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# STABILIZED REVERSING WITH VEHICLE TRAILER COMBINATIONS

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## ABSTRACT

Reversing with a vehicle trailer combination is difficult for unskilled drivers because the trailer has to be guided through permanent steering commands. The kinematics of such a system forces the steering commands to be in the opposite direction compared to driving forwards. A driver reversing can be assisted by controlling the angle between the longitudinal axle of the towing vehicle and the longitudinal axle of the trailer. The set value for the combination's articulation angle depends on the towing vehicle's steering angle, wheelbase, displacement path of the hitch and the trailer's towing bar length. The trailer is steered by lateral motion of the hitch and therefore the trailer's coupling point.

Based on a simple kinematic behaviour model of the combination's articulation angle a simple algorithm to generate the set value is derived. Based on the same model a towing vehicle velocity adaptive controller algorithm is derived using ackermann's formula. The algorithms were successfully tested in a prototype vehicle using rapid control prototyping equipment. As example results of reversing without steering wheel action straight with more than 20 km/h and parts of steady state skidpad testing backwards without corrective actions through the driver are shown.

## 1. INTRODUCTION

For unskilled drivers reversing with a vehicle trailer combination is difficult because permanent steering commands are necessary to guide the trailer. In [1] a model is presented for controller design to stabilize a combination while reversing through active steering. Based on this simple kinematic model a second approach is tested in simulation and road trials using an active hitch. The active hitch allows to displace the trailer's coupling point in lateral direction. Details on further uses of the active hitch according stabilization while forward driving fast and reduction of corner cutting can be found in [2] and [3].

A simple state space ackermann controller is used to stabilize the trailer while reversing around a steering angle dependent articulation angle as target value. The knowledge of the active hitch's displacement, the articulation angle and the combinations velocity are necessary inputs for the controller. As parameter especially the trailer's towing bar length is important. In simulation and road trials the effectiveness of the stabilization is shown.

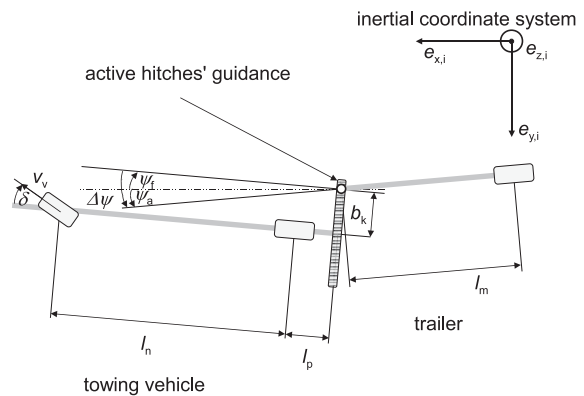
ulation angle and the combinations velocity are necessary inputs for the controller. As parameter especially the trailer's towing bar length is important. In simulation and road trials the effectiveness of the stabilization is shown.

## 2. MODELING

The combination's articulation angle behaviour at low speed is described by a simple kinematic equation [1]. The trailer's motion depends on the trailer's geometry, the towing vehicle's steering wheel angle and the hitch's displacement path. To simplify matters the less dominant dynamic effects of the wheel, of the steering mechanism and further vehicle's degrees of freedom will not be considered. From figure 1 the differential equation for the time dependent behaviour of the articulation angle can be derived.

$$\begin{aligned} \Delta\dot{\psi} = & -v_v \left( \frac{1}{l_m} \cos(\delta) \sin(\Delta\psi) \dots \right. \\ & + \frac{1}{l_n} \left( \frac{l_p}{l_m} \cos(\Delta\psi) + 1 \right) \sin(\delta) \dots \quad (1) \\ & \left. + \dot{b}_k \frac{1}{l_m} \cos(\Delta\psi) \right) \end{aligned}$$

Equating the articulation angle's velocity  $\Delta\dot{\psi}$  and the



**Fig. 1.** Kinematic vehicle trailer combination model  
velocity of the hitch's lateral motion  $\dot{b}_k$  to zero, a steady

Symbol	Description	Dimension
$b$	length $y$ -direction	m
$e$	controller input	diverse
$l$	length $x$ -direction	m
$r$	controller parameter	diverse
$v$	combination's velocity	m/s
$\delta$	steering angle	rad
$\Delta\psi$	articulation angle	rad

**Table 1.** Nomenclature with dimensions for modeling and controller design

state dependency between the steering angle  $\delta$  and the articulation angle  $\Delta\psi$  is derived.

$$\frac{\tan(\delta)}{l_n} = \frac{\sin(\Delta\psi)}{l_p \cos(\Delta\psi) + l_m}. \quad (2)$$

For small deviations of the steering and the articulation angle around the operating point  $0^\circ$  equation 1

$$\Delta\dot{\psi} = \frac{-v_v}{l_m} \Delta\psi + \frac{1}{l_m} \dot{b}_k - \frac{v_v}{l_n} \left( \frac{l_p}{l_m} + 1 \right) \delta \quad (3)$$

and equation 2

$$\Delta\psi = \frac{l_p + l_m}{l_n} \delta \quad (4)$$

are linearized by execution of a Taylor approximation of degree one. Equation 4 will be used as base for the calculation of the target value for the articulation angle while reversing.

### 3. CONTROLLER DESIGN

While reversing the vehicle trailer combination is stabilized by controlling the trailer – similar to a standing pendulum – around an instable idle point given by equation 2. In [1] a simple model to design a controller to control the articulation angle around a target value using the steering angle. The model was upgraded with the active hitch's degree of freedom lateral motion. Based on the upgraded model a controller is designed to control the articulation angle around a given target value. Using the linearized equation 3 a controller is designed for the lateral motion  $\dot{b}_k$  as actuating variable and the articulation angle  $\Delta\psi$  as control variable. Interpreting the actuating variable as additional state to include the lateral motion into the controller design, results the following state space model

$$\dot{\mathbf{x}}(t) = \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) \quad (5)$$

$$\begin{Bmatrix} \Delta\dot{\psi} \\ \dot{b}_k \end{Bmatrix} = \begin{bmatrix} \frac{-v_v}{l_m} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \Delta\psi \\ b_k \end{Bmatrix} + \begin{Bmatrix} \frac{1}{l_m} \\ 1 \end{Bmatrix} \dot{b}_k. \quad (6)$$

As the controllability matrix

$$\mathbf{Q}_{s,rueck} = \begin{bmatrix} \frac{1}{l_m} & \frac{-v_v}{l_m^2} \\ 1 & 0 \end{bmatrix} \quad (7)$$

shows, the system is controllable for vehicle speeds  $v_v \neq 0$ . Because the system is controllable and has one actuating variable  $\dot{b}_k$ , using ackermann's formula according [4] a state space controller can be designed. In this approach it is to be expected that the combination's articulation angle  $\Delta\psi$  and the lateral displacement path of the active hitch  $b_k$  can be measured. The eigenvalues can be calculated following equation

$$\iota = \det[s \mathbf{I} - (\mathbf{A} - \mathbf{B} \mathbf{R})]. \quad (8)$$

As the system owns to states, two poles

$$\mathbf{p} = [p_1 \quad p_2]^T \quad (9)$$

have to be placed for the closed loop system. It follows from the above

$$\mathbf{r}_{b_k,rueck} = \begin{Bmatrix} -l_m \left( p_1 + p_2 + \frac{v_v}{l_m} + p_1 p_2 \frac{l_m}{v_v} \right) \\ p_1 p_2 \frac{l_m}{v_v} \end{Bmatrix} \quad (10)$$

as velocity dependent feedback law. The target value of the lateral motion's speed  $\dot{b}_k$  is to be calculated as follows

$$\dot{b}_{k,soll,rueck} = \mathbf{r}_{b_k,rueck} \begin{Bmatrix} e_{\Delta\psi} \\ e_{b_k} \end{Bmatrix}. \quad (11)$$

As the mechanical limitation of the hitch's lateral motion can be included easily by a limitation of the hitch's lateral displacement path's target value, the velocity of the lateral motion's target value is integrated

$$b_{k,soll,rueck} = \int_{t_0}^t \dot{b}_{k,soll,rueck}(\tau) d\tau \quad (12)$$

to calculate the target value of the hitch's lateral displacement path.

### 4. STABILIZING WHILE REVERSING IN SIMULATION

As previously shown in equation 7 a combination is fully controllable during reversing slowly based on knowledge of the articulation angle  $\Delta\psi$  and the speed of the hitch's lateral motion  $\dot{b}_k$ . While reversing a combination behaves like nonlinear instable system – standing pendulum – it is necessary to control the articulation angle for gaining a stabilized system. The dominant effects result from the combination's geometry and the passed distance. The dynamic behaviour dominating parameters like moments of inertia and masses will not be considered.

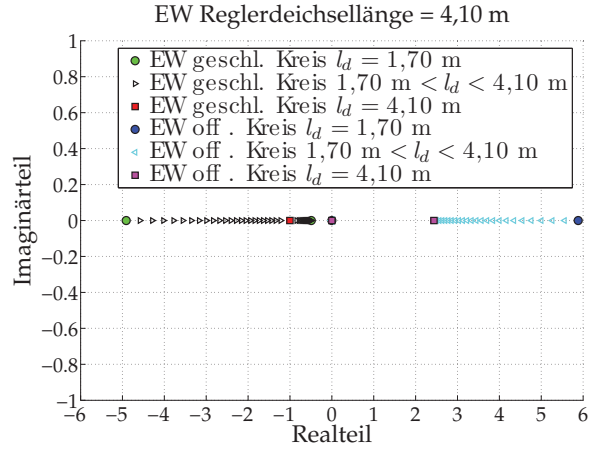
As a first approach the poles of the closed loop system both poles are placed at  $p_1 = p_2 = -1$ . Knowing the towing vehicle's velocity  $v_v$ , the combination's geometry and the poles with equation 10 the velocity dependent controller parameters can be calculated. This controller stabilizes the trailer around a target value for the articulation angle. In the first place the articulation angle's target value will be calculated based on equation 4 for small steering and articulation angles. The most important previously unknown parameter is the distance from the coupling point till the trailer's centre of rotation. The most important nonlinearity for stabilizing while reversing is the limitation of the active hitch's lateral motion. The necessary force and velocity for the lateral motion do not play a dominant role and therefore they are not considered for further investigations.

Following is shown, how a combination can be stabilized while reversing, what effects a parameter's  $l_d$  – towing bar length – variation has concerning the controlling results and how a steady control deviation can be avoided using an I" part for calculating the target value. Additionally limitations of the stabilization while reversing are shown.

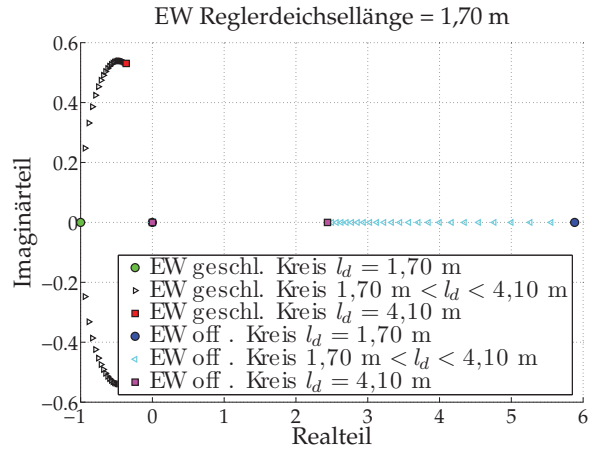
Figure 2 (a) shows the sensitivity of the open and the closed loop control according a variation of the towing bar length  $l_d$ . For small towing bar lengths the combination is more unstable, because even small displacements force large articulation angles. The controller parameters are calculated with a towing bar length of 4, 10 m and both poles are placed at  $-1$ . The larger the deviation between the trailers' towing bar length and the controller assumed is, the more unstable the closed loop control gets. If the controller assumed towing bar length is smaller than the trailer's one, demonstrated in figure 2 (b), the closed loop circuit gets oscillatory and more unstable.

Figure 3 (a) shows the behaviour of an ideal adjusted controller for diverse combination velocities following a steering impuls as excitation. The most important state is the active hitch's displacement path. Because the displacement path and the lateral displacement velocity are limited, the combination can only be stabilized up to a maximum velocity. The faster the combination is driven the more displacement path and is necessary to stabilize the trailer, as there is more trailer displacement in less time to be corrected. The time response to a constant frequency of steering impulses is demonstrated in figure 3 (a). The combination is stable and oscillates with a constant frequency. Because of this weaker excitation the combination can be stabilized up to  $v_v = -36 \text{ km/h}$

The active hitch's reaction due to a lane change while reversing is shown in figure 4 (a). Also for a lane change it gets more difficult to stabilize the combination at higher velocities. The countersteering while lane change aids the stabilization. Figure 4 (b) demon-



(a) Towing bar length = 4, 10 m

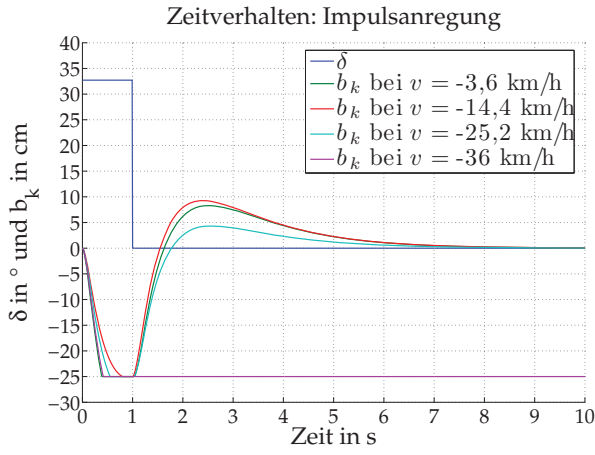


(b) Towing bar length = 1, 70 m

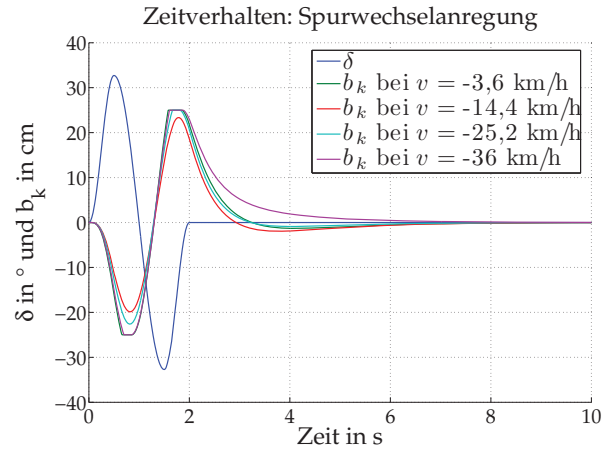
**Fig. 2.** Sensitivity analysis of eigenvalues according to variation of towing bar length with constant velocity  $v = -36 \text{ km/h}$  and both closed loop system poles at  $p_1 = -1$  und  $p_2 = -1$

strates the behaviour at steady-state skid pad testing and different velocities while reversing. The active hitch is only able to stabilize the combination up to a maximum velocity, because of the displacement path limitation. But also the steady-state skidpad testing backwards can be driven till this maximum velocity.

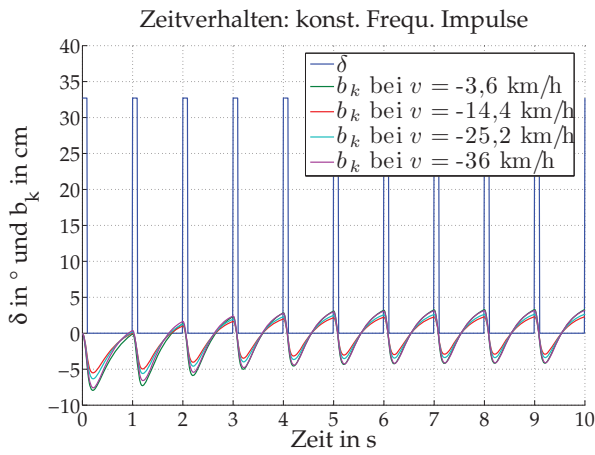
The equation to calculate the displacement velocity as controller output shows that for example steady-state skidpad testing backwards and large angles lead to steady displacement of the active hitch. The displacement of the active hitch should be avoided because a maximum displacement path to stabilize the combination should be available. Figure 5 shows the steady control deviation and the result of an approach. The deviation of the target value is integrated and added to the controller input. A steering wheel angle of  $360^\circ$  leads to a controller deviation according the displacement path of about 18 cm. The necessary displace-



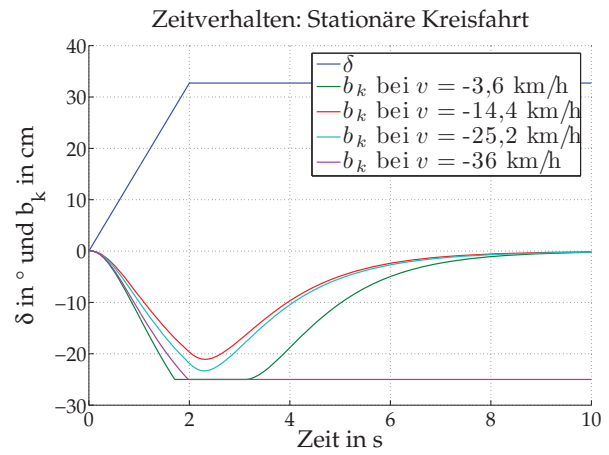
(a) Steering impulse



(a) Lane change



(b) Constant frequency exciting steering impulses



(b) Steady-state skidpad testing backwards

**Fig. 3.** Closed loop system time response – poles at  $p_1 = -1$  und  $p_2 = -1$  – for different velocities

**Fig. 4.** Closed loop system time response – poles at  $p_1 = -1$  und  $p_2 = -1$  – for different velocities

ment path becomes smaller using an integral component for the controller, the articulation angle time response shows nearly the same behaviour.

The simulation results show that a combination can be stabilized with an active hitch while driving backwards if the combination's geometry, the articulation angle and the displacement path. For the controller algorithm the towing bar length should be known. The faster the active hitch reacts, the minor displacement path is needed to stabilize the combination. The poles are placed as compromise between noise sensitivity and compensation velocity. The stabilization will be proved in road trials based on the simulation results.

## 5. STABILIZING WHILE REVERSING IN ROAD TRIALS

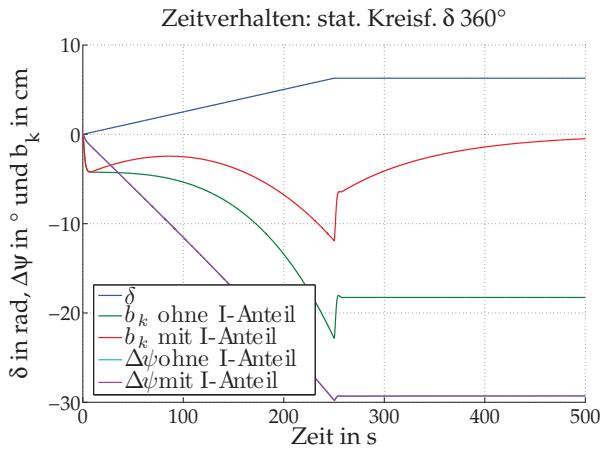
Reversing is difficult for unskilled drivers. Especially path planning is tricky and the slightest disturbance leads to a sort of jackknifing of the trailer. Without active

hitch the combination has to be stabilized through steering. A position controlled active hitch enables stabilization of combinations as chapter 4 shows. Road trials are performed and there results will be presented in the following. The controller design was discussed in chapter 3.

Equation 10 shows the controllers' velocity dependence. The most important controller parameter is the trailer's towing bar length. The trailer's mass and moment of inertia will not be considered. Here towing bar length describes the distance between the trailer's centre of rotation and the coupling point at the vehicle. The poles are placed at  $p_1 = p_2 = -1$ . The controller will be tested without integral component in road trials. Based on the combination's geometry equation 4 is used to calculate the steering angle dependent articulation angle's target value.

Figure 7 (a) demonstrates how a driver has to stabilize the trailer through steering while reversing straight. Explicit steering intervention is necessary to prevent



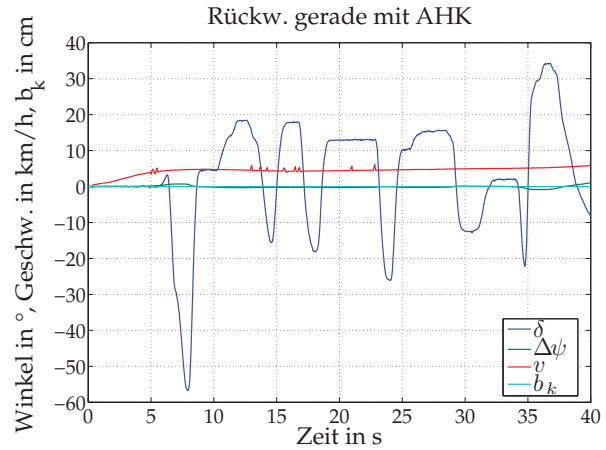


**Fig. 5.** closed loop system time response – poles at  $p_1 = -1$  und  $p_2 = -1$  – for steady-state skidpad testing backwards with constant steering angle and controller with and without integral component

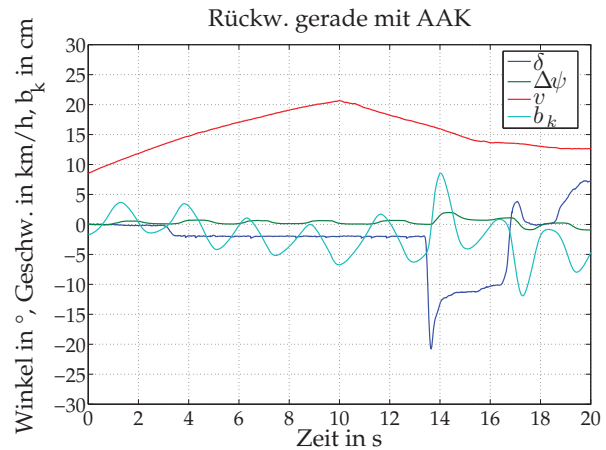
the trailer's jackknifing. The shown driving manoeuvre forces steering wheel angles  $\geq 10^\circ$ . Without the steering intervention the trailer would have jackknifed up to the towing bar contacts the rear bumper. The combination's driving behaviour with active hitch is demonstrated in figure 7 (b). The first 13,5 s the combination reverses nearly straight. The articulation's course shows the trailer's jackknifing. As soon as the trailer jackknifes the combination is stabilized through a controlled displacement of the active hitch. Because the combination is constantly disturbed, permanent stabilizing controller interventions are necessary. From second 13,5 the combination is steered on a light curve, therefore an articulation angle is given as target value the trailer is stabilized around. While reversing a maximum velocity over 20 km/h is driven and the active hitch is able to stabilize the combination. To stabilize the combination a displacement path about 5 cm is necessary.

The active hitch enables the combination to follow a stable skid pad while reversing. Due to the limitation of the displacement path the combination cannot be steered jerkily on the desired skid pad. The active hitch's behaviour is shown in figures 7 (a) and (b) for arbitrary driving manoeuvres. The left side shows in detail the control of the articulation angle's target value. The right side demonstrate the effects an imprecise chosen parameter for the towing bar length. The necessary displacement path becomes larger.

With an active hitch reversing straight with comparatively high velocities can be achieved without trailer jackknifing. One driving manoeuvre with a velocity close to 30 km/h is demonstrated in figure 8 (a). The necessary displacement path becomes larger, therefore there is a maximum velocity up to that a combination can be stabilized. The maximum velocity was not tested



(a) conventional hitch



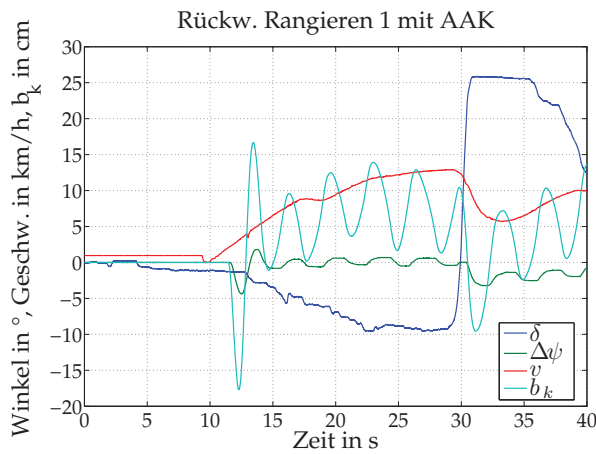
(b) active hitch

**Fig. 6.** Combination behaviour while reversing

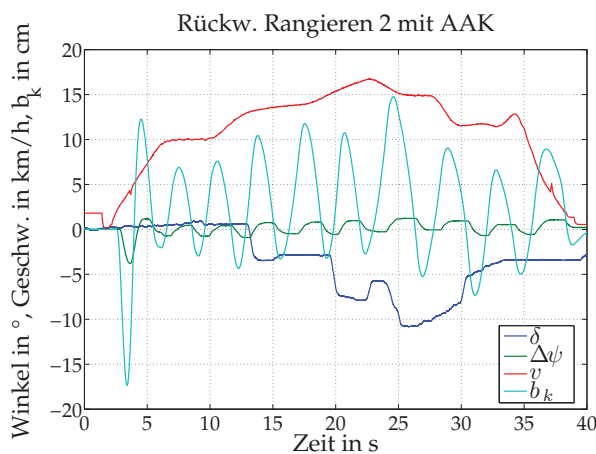
due to safety reasons. If the trailer jackknifes at that high velocities the braking distance becomes too large and the jackknifing will be that way, that trailers' towing bar and the towing vehicle collide. Are the controller inputs offset afflicted, the active hitch's displacement path will show an offset. In this case the controller stabilizes around displacement path's zero position but the adjusted displacement offset. In figure 8 (b) this reaction is shown. Bei nahezu gerader Rückwärtsfahrt pendelt die Anhängerkupplungsposition nicht um die Nullstellung. Kleine Offsets sind zum einen in der Messung des Lenkradwinkels und zum anderen in der Messung des Gespannknickwinkels möglich.

## 6. SUBSUMPTION

With an active hitch vehicle trailer combinations can be stabilized while reversing. The active hitch's stabilization functionality simplifies reversing. Unskilled drivers find a more easy to estimate driving behaviour of the combination. Counter steering for stabilization



(a) Reversing with beginning skid pad driving



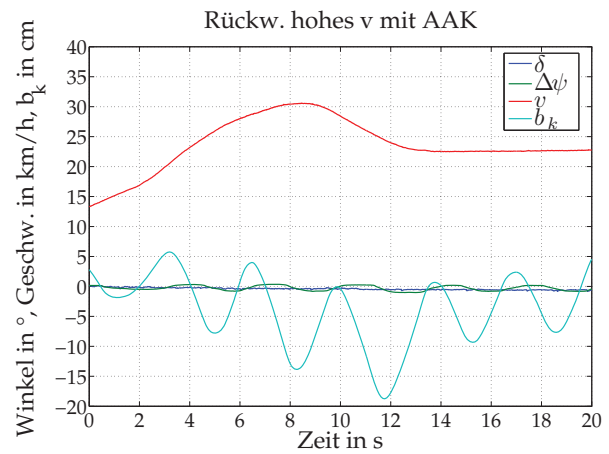
(b) Imprecise controller chosen towing bar length

**Fig. 7.** Combination behaviour while reversing

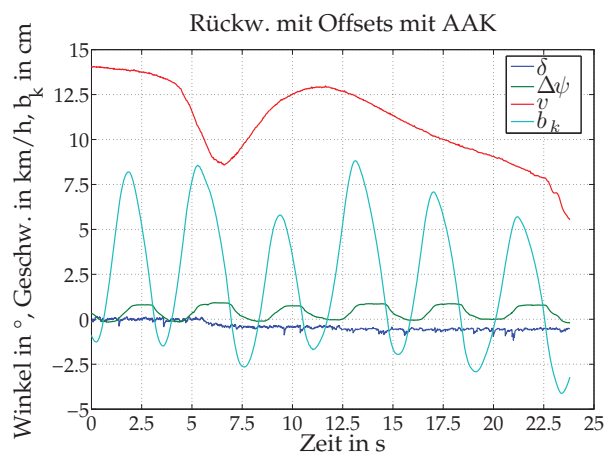
is not necessary. The combination can follow the driver planned path without steering intervention for stabilization. Especially reversing straight simplifies manoeuvring considerably. For good controlling results the combinations geometry has to be known. Controller inputs are the articulation angle, the combination's velocity and the steering angle. Offsets of the controller inputs and deviations of the controller parameters according to the real combination's geometry lead to poor stabilizing results. On the one hand an offset in displacement path occurs, on the other hand the necessary displacement path for stabilization becomes major. The most dominant controller parameter to be adjusted is the towing bar length.

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(a) Reversing straight with high velocity



(b) Reversing with controller offset

**Fig. 8.** Combination behaviour while reversing straight

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